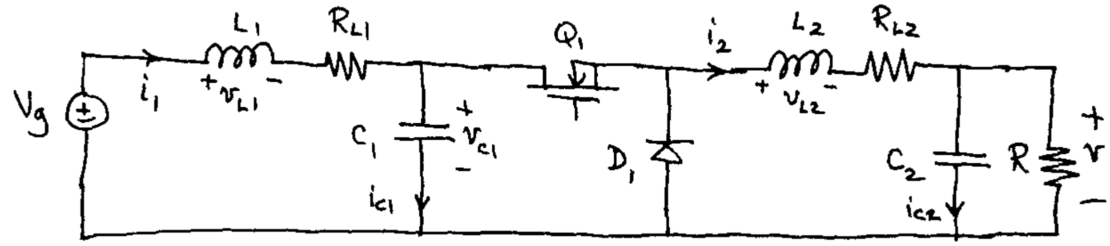
R.W. Erickson
Fundamentals of
Power Electronics

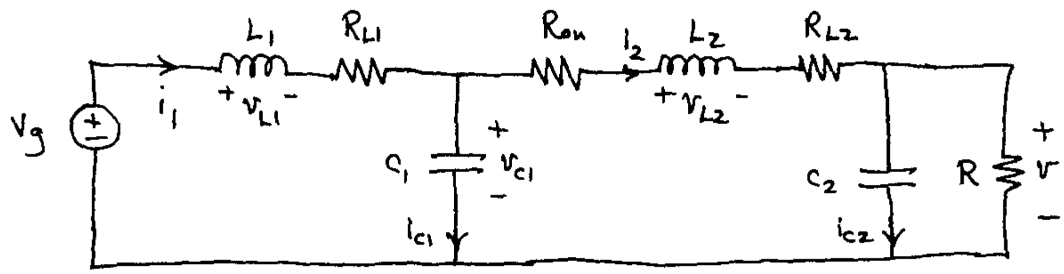
Solution to Problem 3.3

Converter circuit, Fig. 3.30:



Model inductor winding resistances R_{L1} and R_{L2}, MOSFET on-resistance R_{on}, and diode forward voltage drop (constant voltage V_D plus voltage across effective resistance R_D).

Subinterval 1 $\quad 0 < t < DT_s$. $\quad Q_1$ on, D_1 off



Inductor voltages and capacitor currents, expressed as functions of quantities that have small ripple:

$$v_{L1}(t) = V_g - i_1(t) R_{L1} - v_{C1}(t)$$

$$v_{L2}(t) = v_{C1}(t) - i_2(t) R_{on} - i_2(t) R_{L2} - v(t)$$

$$i_{C1}(t) = i_1(t) - i_2(t)$$

$$i_{C2}(t) = i_2(t) - v(t)/R$$

Small-ripple approximation: $i_1(t) \approx I_1$, $i_2(t) \approx I_2$, $v_{c1}(t) \approx V_{c1}$, $v(t) \approx V$

we get

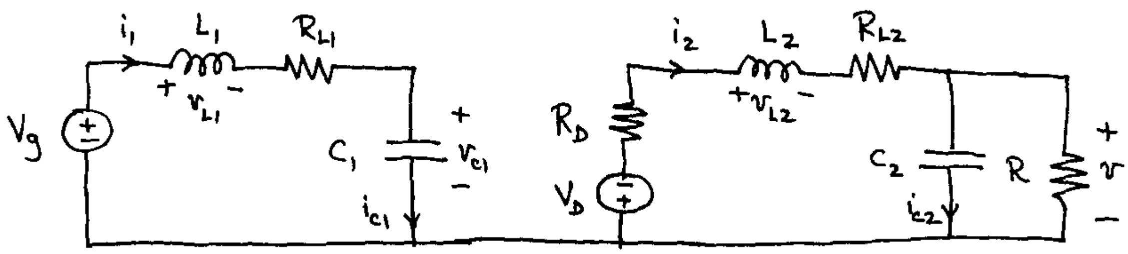
$$v_{L1}(t) \approx V_g - I_1 R_{L1} - V_{c1}$$

$$v_{L2}(t) \approx V_{c1} - I_2 R_{on} - I_2 R_{L2} - V$$

$$i_{c1}(t) \approx I_1 - I_2$$

$$i_{c2}(t) \approx I_2 - V/R$$

Subinterval 2 $\qquad DT_s < t < T_s \qquad Q_1$ off, D_1 on



$$v_{L1}(t) = V_g - i_1(t) R_{L1} - v_{c1}(t)$$

$$v_{L2}(t) = -V_D - i_2(t) R_D - i_2(t) R_{L2} - v(t)$$

$$i_{c1}(t) = i_1(t)$$

$$i_{c2}(t) = i_2(t) - v(t)/R$$

Small ripple approximation:

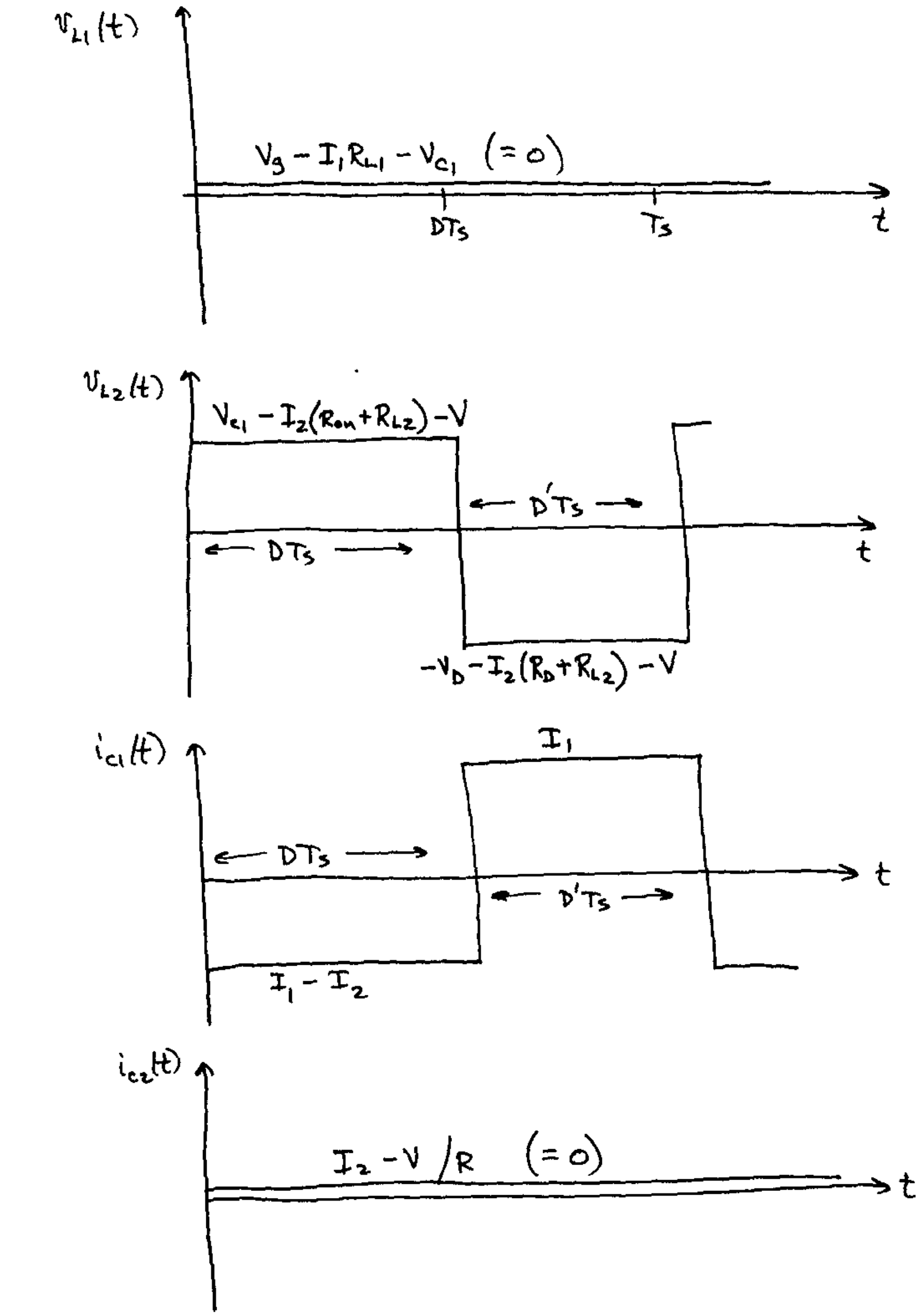
$$v_{L1}(t) \approx V_g - I_1 R_{L1} - V_{c1}$$

$$v_{L2}(t) \approx -V_D - I_2 R_D - I_2 R_{L2} - V$$

$$i_{c1}(t) \approx I_1$$

$$i_{c2}(t) \approx I_2 - V/R$$

Switched waveforms:



Equate average inductor voltages and capacitor currents to zero:

$$\langle v_{L1}(t)\rangle = 0 = V_g - I_1 R_{L1} - V_{C1}$$

$$\langle v_{L2}(t)\rangle = 0 = D\left[V_{C1} - I_2(R_{on}+R_{L2}) - V\right] + D'\left[-V_D - I_2(R_D+R_{L2}) - V\right]$$

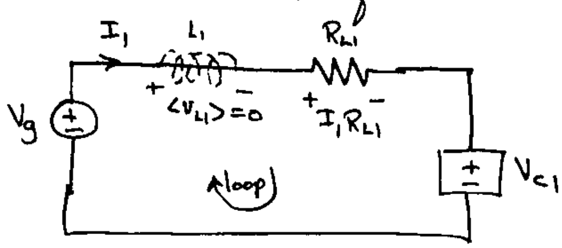
$$\langle i_{C1}(t)\rangle = 0 = D\left[I_1 - I_2\right] + D'\left[I_1\right]$$

$$\langle i_{C2}(t)\rangle = 0 = I_2 - V/R$$

Derive equivalent circuit

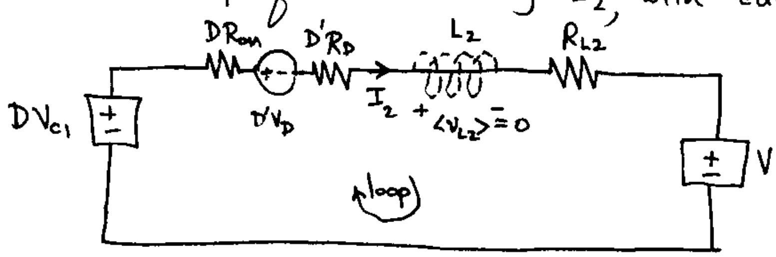
Inductor L_1: $\langle v_{L1}\rangle = 0 = V_g - I_1 R_{L1} - V_{C1}$

A loop equation containing L_1, with current I_1



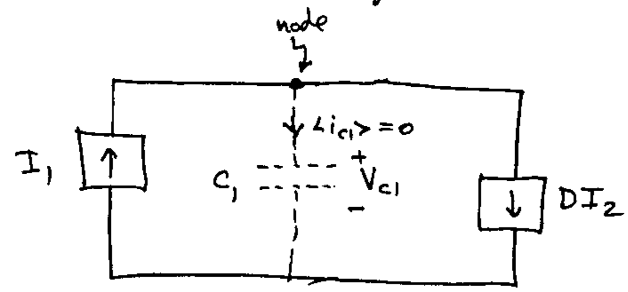
Inductor L_2: $\langle v_{L2}\rangle = 0 = DV_{C1} - DR_{on}I_2 - D'R_D I_2 - R_{L2}I_2 - V - D'V_D$

A loop equation containing L_2, with current I_2



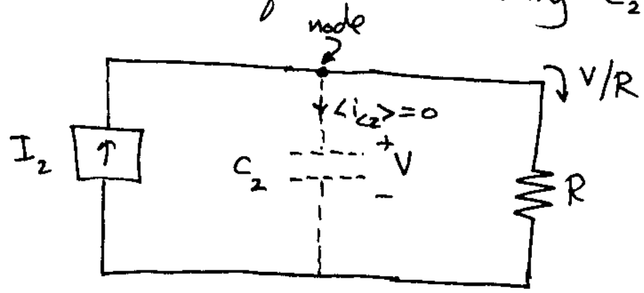
Capacitor C_1 : $\langle i_{c1} \rangle = 0 = I_1 - D I_2$

A node equation containing C_1, with voltage V_{c1}

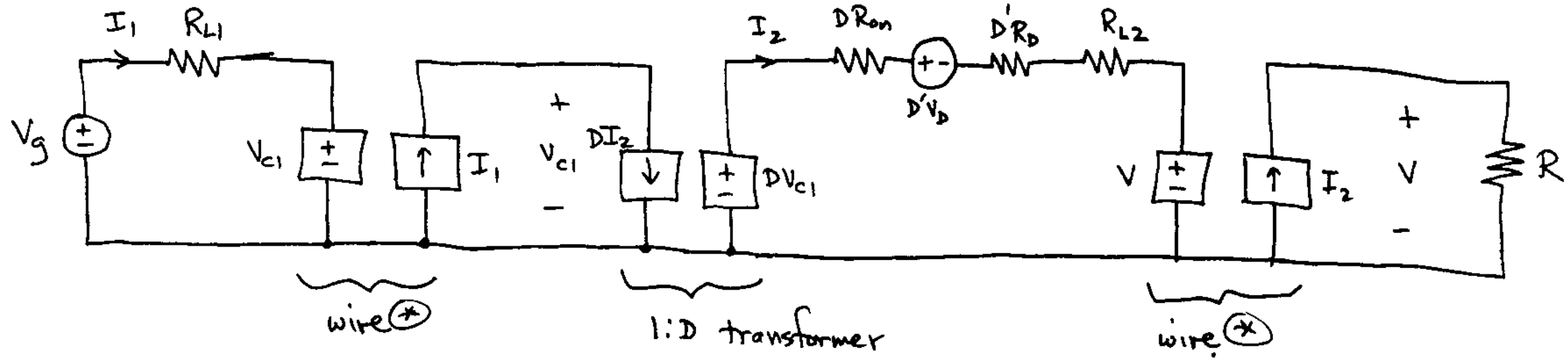


Capacitor C_2 : $\langle i_{c2} \rangle = 0 = I_2 - V/R$

A node equation including C_2, with voltage V.

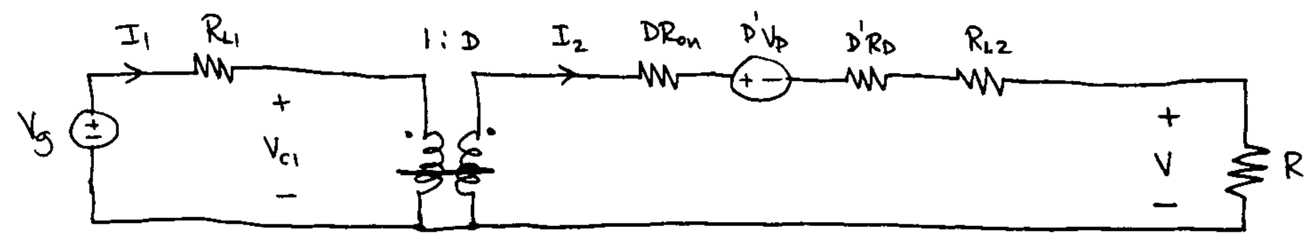


write circuits together:



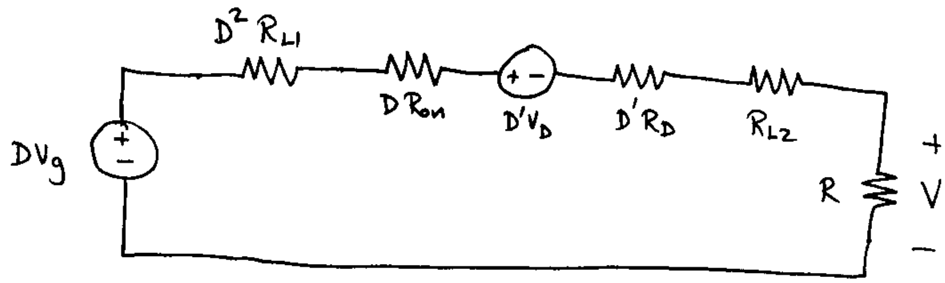
(*) Note that 1:1 transformers are not needed in these locations — a direct connection will suffice! In fact, in the actual converter there is no switching at these points. Inductor L_1 is directly connected to C_1, and L_2 is always connected to C_2

The DI_2 and DV_{C1} dependent sources form a 1:D effective dc transformer.

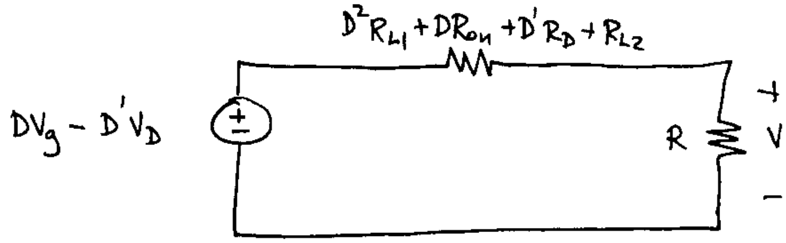


b) Solve model to find V

Push V_g and R_{L1} through dc transformer;



Combine elements;



Voltage divider formula;

$$\boxed{V = \left(DV_g - D'V_D\right) \frac{R}{R + D^2 R_{L1} + DR_{on} + D'R_D + R_{L2}}}$$

c) Derive an expression for efficiency η. Manipulate into form similar to Eq. (3.35)

From the equivalent circuit on page 6,

$$P_{in} = V_g I_1$$

$$P_{out} = V I_2$$

and $$I_1 = D I_2$$

So

$$\eta = \frac{P_{OUT}}{P_{in}} = \frac{V\, I_2}{V_g\, I_1} = \frac{1}{D}\,\frac{V}{V_g}$$

From part (b),

$$\frac{V}{V_g} = \left(D - D'\,\frac{V_D}{V_g}\right)\frac{R}{R + D^2 R_{L1} + D R_{on} + D' R_D + R_{L2}}$$

(note that it is OK for the right side of the equation to depend on V_g).

So

$$\eta = \left(1 - \frac{D'}{D}\,\frac{V_D}{V_g}\right)\frac{1}{\left(1 + D^2\frac{R_{L1}}{R} + D\frac{R_{on}}{R} + D'\frac{R_D}{R} + \frac{R_{L2}}{R}\right)}$$

This is a good design-oriented way to express the efficiency, because it exposes how each loss element reduces the efficiency. The effect of the diode voltage drop V_D is expressed in terms of V_g, while the loss resistances are compared to R. The various losses also depend on duty cycle.