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Fundamentals of
Power Electronics

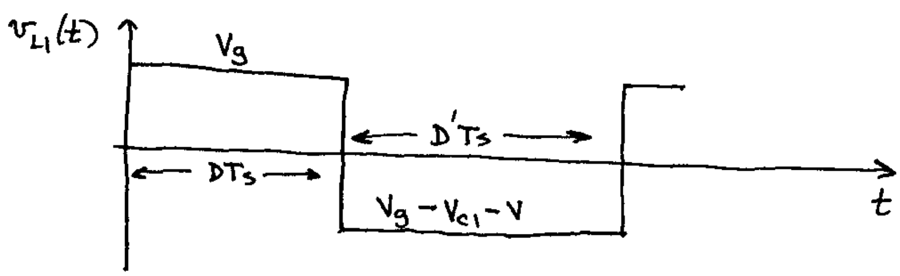
Solution to Problem 2.3

See solution to problem 2.2 for subinterval circuits, equations, small ripple approximation, capacitor current and inductor voltage waveforms, and solution for dc voltages and currents.

a) Derive expressions for inductor current ripples and capacitor voltage ripples

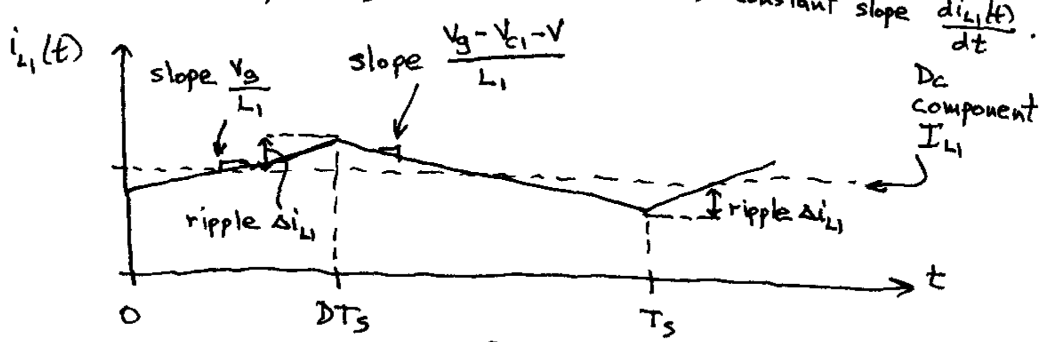
Inductor L_1

Voltage waveform (from problem 2.2):



Slope of inductor current is found using $L_1 \dfrac{di_{L1}(t)}{dt} = v_{L1}(t)$

so $\dfrac{di_{L1}(t)}{dt} = \dfrac{v_{L1}(t)}{L_1}$. [During a given subinterval, the voltage $v_{L1}(t)$ is essentially constant $\Rightarrow$ constant slope $\dfrac{di_{L1}(t)}{dt}$.



Peak ripple (i.e., peak-to-average) $= \Delta i_{L1}$

peak-to-peak ripple $= 2\Delta i_{L1}$

During first subinterval, $i_{L1}(t)$ changes from $(I_{L1} - \Delta i_{L1})$ to $(I_{L1} + \Delta i_{L1})$, for a total change of $2\Delta i_{L1}$.

$$\left(\text{change in } i_{L1}(t)\right) = (\text{slope})(\text{time})$$

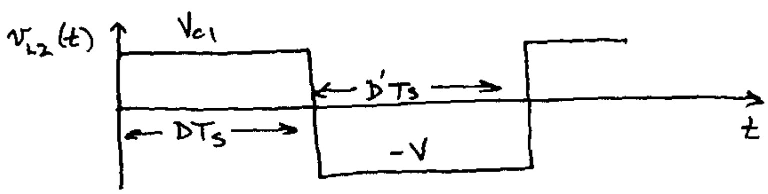
$$\left(2\Delta i_{L1}\right) = \left(\frac{V_g}{L_1}\right)\left(DT_s\right) \qquad \text{for first subinterval}$$

so $$\boxed{\Delta i_{L1} = \frac{V_g DT_s}{2L_1}}$$

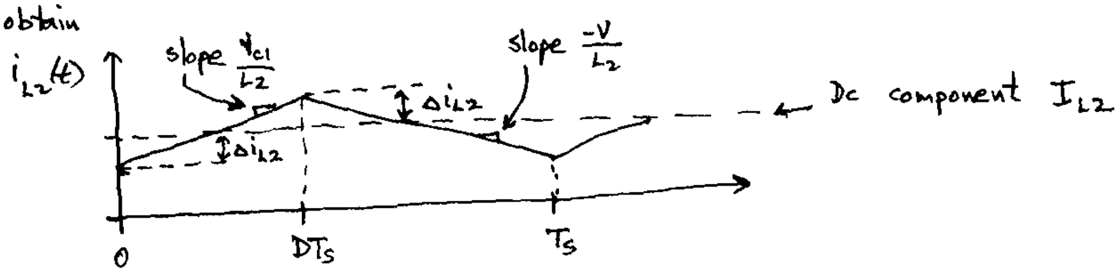
The same result could be derived via a similar analysis of subinterval 2.

<u>Inductor L_2</u>

Voltage waveform, from problem 2.2:



so we obtain



First subinterval:

$$\underbrace{2\Delta i_{L2}}_{\text{change in } i_{L2}(t)} = \underbrace{\left(\frac{V_{C1}}{L_2}\right)}_{\text{slope}} \underbrace{\left(DT_s\right)}_{\text{time}}$$

solve for Δi_{L2}:

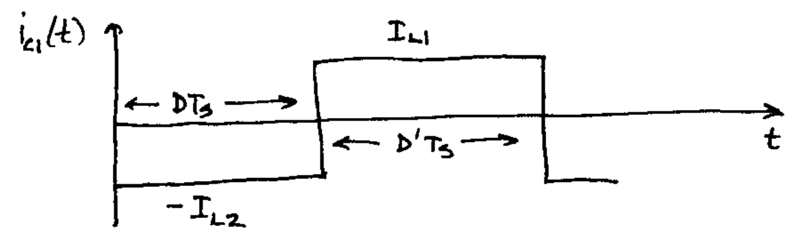
$$\Delta i_{L2} = \frac{V_{C1}\, DT_s}{2 L_2}$$

In problem 2.2, it was found that $V_{C1} = V_g$. So

$$\boxed{\Delta i_{L2} = \frac{V_g\, DT_s}{2 L_2}}$$

Capacitor C_1

Current waveform, from problem 2.2:



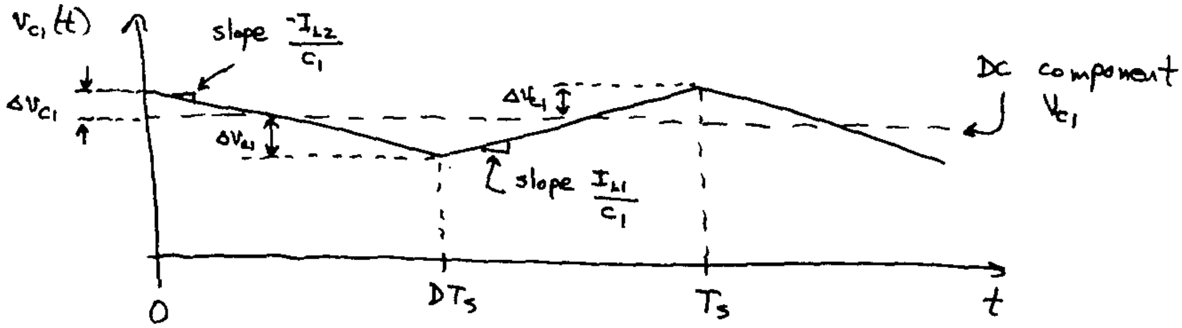
so the capacitor voltage waveform is found using the relationship $i_{C1}(t) = C_1 \dfrac{dv_{C1}(t)}{dt}$

$$\text{Slope } \frac{dv_{C1}(t)}{dt} = \frac{i_{C1}(t)}{C_1}$$

During a given subinterval, the current $i_{C1}(t)$ is essentially constant $\Rightarrow$ constant slope.

Slope during first subinterval $0 < t < DT_s$ is $-\dfrac{I_{L2}}{C_1}$

Slope during second subinterval $DT_s < t < T_s$ is I_{L1}/C_1



During the first subinterval, $v_{c1}(t)$ changes by

$$\underbrace{\left(-2\,\Delta v_{c1}\right)}_{\substack{\text{change}\\ \text{during}\\ 0<t<DT_s}} = \underbrace{\left(-\frac{I_{L2}}{C_1}\right)}_{\text{slope}}\underbrace{\left(DT_s\right)}_{\text{time}}$$

Solve for Δv_{c1}:

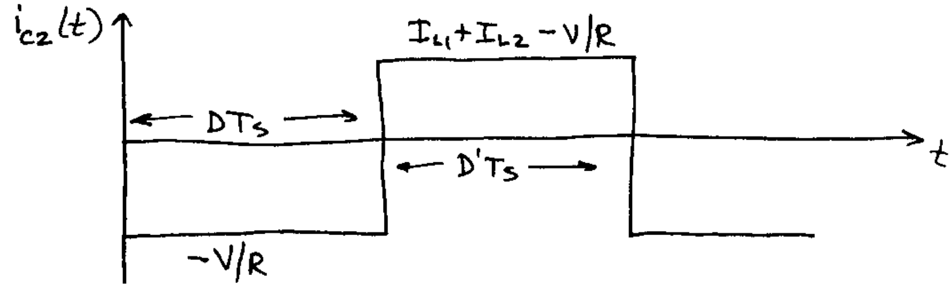
$$\Delta v_{c1} = \frac{I_{L2}\,DT_s}{2\,C_1}$$

Substitute solution for I_{L2} (from problem 2.2): $I_{L2} = \dfrac{D}{(D')^2}\,\dfrac{V_g}{R}$

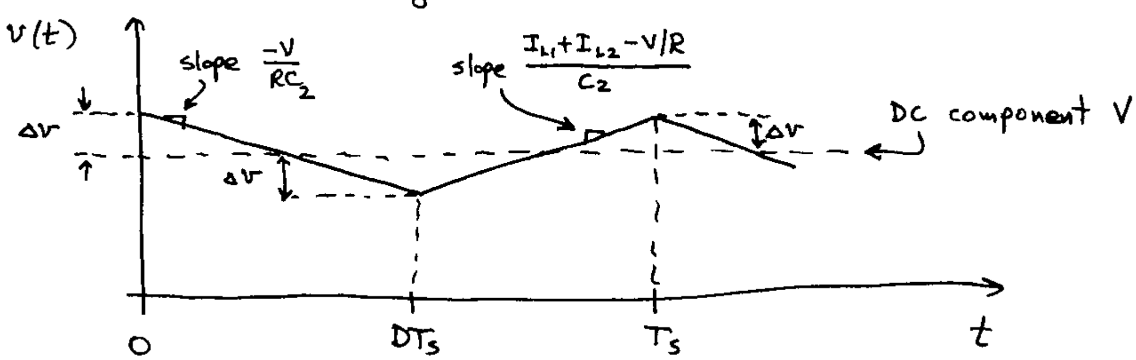
$$\boxed{\Delta v_{c1} = \left(\frac{D}{D'}\right)^2 \frac{V_g T_s}{2RC_1}}$$

Capacitor C_2

Current waveform $i_{C2}(t)$, from problem 2.2:



So the capacitor voltage waveform is



Change in capacitor voltage during first subinterval is

$$(-2\Delta v) = \left(-\frac{V}{RC_2}\right)(DT_s)$$

$$\Rightarrow \Delta v = \frac{VDT_s}{2RC_2}$$

Substitute solution for V from problem 2.2: $V = \frac{D}{D'} V_g$

$$\boxed{\Delta v = \frac{D^2}{D'} \frac{V_g T_s}{2RC_2}}$$

b) Sketch the waveforms of the transistor voltage $v_{DS}(t)$ and transistor current $i_D(t)$, and give expressions for their peak values.

Again, refer to schematic and waveforms in solution to problem 2.2. The transistor current $i_D(t) = i_{Q_1}(t)$ is sketched in the problem 2.2 solution.

The transistor drain-to-source voltage $v_{DS}(t)$ is equal to approximately zero during the first subinterval, when the transistor conducts. During the second subinterval, the transistor is off and the diode conducts. The converter circuit is then as sketched on p.3 of the prob. 2.2 solution. It can be seen that

$$(*) \qquad v_{DS}(t) = v_{c1}(t) + v(t) \qquad \text{for subinterval 2}$$

small ripple approximation

$$v_{DS}(t) \approx V_{c1} + V$$

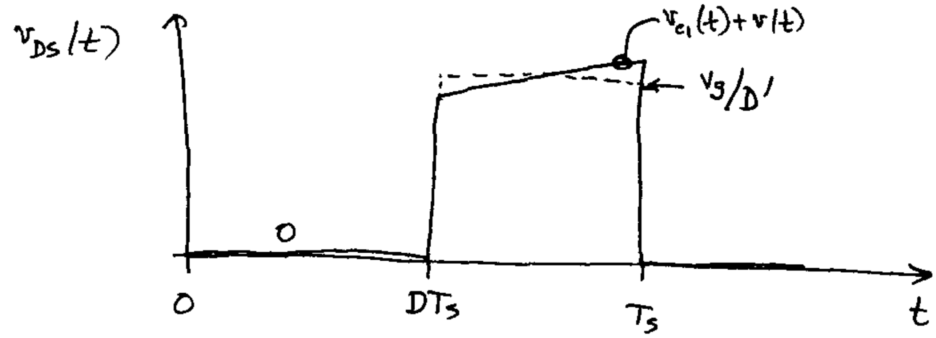
$$= V_g + \frac{D}{D'} V_g \qquad \text{using solution for } V_{c1} \text{ and } V$$

$$= V_g \left(\frac{D' + D}{D'} \right) \qquad \text{note } D' + D = 1$$

$$= \frac{1}{D'} V_g$$

[A caveat: Eq. (*) above is expressed in terms of quantities that have small ripple: $v_{c1}(t)$ and $v(t)$. We could have instead written $v_{DS}(t) = V_g - v_{L1}(t)$, but the result would not be useful because the switching ripple in $v_{L1}(t)$ is not small and cannot be ignored!

So the transistor voltage waveform is



End of problem 2.3