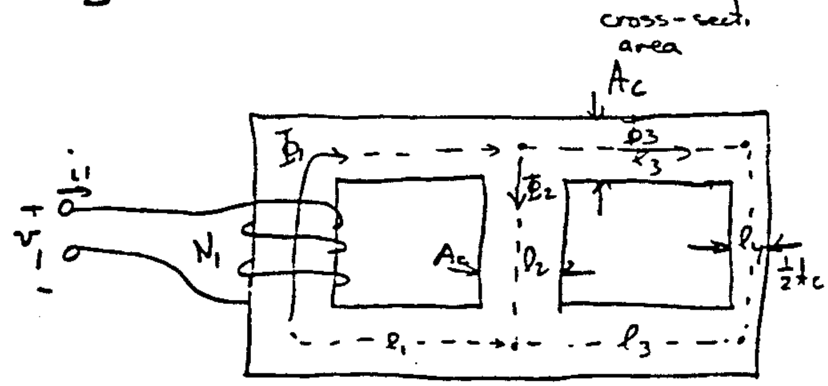
Solution to Problem 12.1

Magnetic Circuits

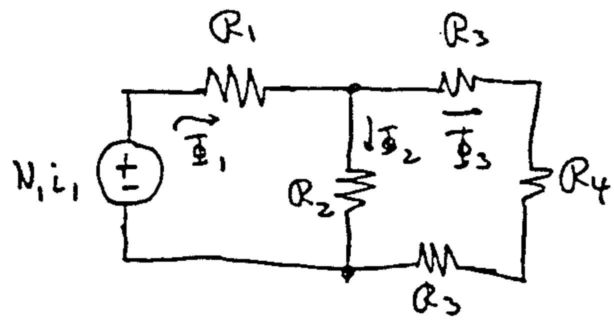


$\mu = 1000\mu_0$

$\mu_0 = 4\pi \cdot 10^{-7}$ H/m

$N_1 = 10$ turns

(a) Equivalent magnetic circuit



where

$$\mathcal{R}_1 = \frac{\ell_1}{\mu A_c} = \frac{(9\text{cm})\ (100\text{ cm/m})}{(4\pi \cdot 10^{-4}\text{H/m})\ (1\text{cm}^2)} = 7.16 \cdot 10^5\ \text{H}^{-1}$$

$$\mathcal{R}_2 = \frac{\ell_2}{\mu A_c} = \frac{(3\text{cm})\ (100\text{cm/m})}{(4\pi \cdot 10^{-4}\text{H/m})(1\text{cm}^2)} = 2.39 \cdot 10^5\ \text{H}^{-1}$$

$$\mathcal{R}_3 = \frac{\ell_3}{\mu A_c} = \mathcal{R}_2 = 2.39 \cdot 10^5\ \text{H}^{-1}$$

$$\mathcal{R}_4 = \frac{\ell_4}{\mu \cdot \frac{1}{2}A_c} = \frac{(3\text{cm})(100\text{ cm/m})}{(4\pi \cdot 10^{-4}\text{H/m})(0.5\text{cm}^2)} = 4.77 \cdot 10^5\ \text{H}^{-1}$$

Total reluctance of right leg

$$\mathcal{R}_r = 2\mathcal{R}_3 + \mathcal{R}_4 = 9.56 \cdot 10^5 \ H^{-1}$$

b) Determine inductance of winding

1^{ST}, determine Φ_1. From the magnetic circuit, we have

$$\Phi_1 = \frac{N_1 i_1}{\mathcal{R}_1 + (\mathcal{R}_2 \| \mathcal{R}_r)}$$

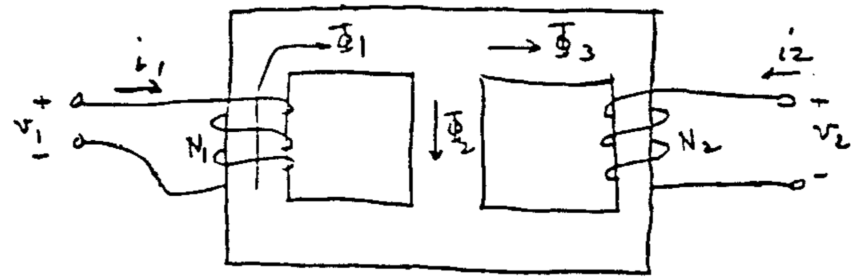
From Faraday's law,

$$v_1 = N_1 \frac{d\Phi_1}{dt} = \frac{N_1^2}{\mathcal{R}_1 + (\mathcal{R}_2 \| \mathcal{R}_r)} \frac{di_1}{dt}$$

Hence, $v_1 = L_1 \dfrac{di_1}{dt}$ with $L_1 = \dfrac{N_1^2}{\mathcal{R}_1 + (\mathcal{R}_2 \| \mathcal{R}_r)}$

$$\Rightarrow L_1 = \frac{(10)^2}{(7.16 \cdot 10^5) + (2.39 \cdot 10^5) \| (9.56 \cdot 10^5)} = 110 \ \mu H$$

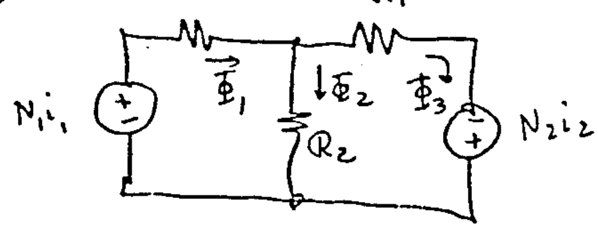
A second winding is added;



$N_2 = 20$ turns

Note direction of wire in second winding, and defined direction of i_2. Just as i_1 tends to increase Φ_1, i_2 tends to increase (rather than decrease Φ_3. The magnetic circuit is

(c)



note polarity of magnetomotive force N.

(d) Express electrical equations for this magnetic circuit in the form

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

so $\quad v_1 = N_1 \dot{\Phi}_1 = L_{11} \dot{i}_1 + L_{12} \dot{i}_2 \qquad$ (1st line)

$$\Rightarrow \Phi_1 = \frac{L_{11}}{N_1} i_1 + \frac{L_{12}}{N_1} i_2$$

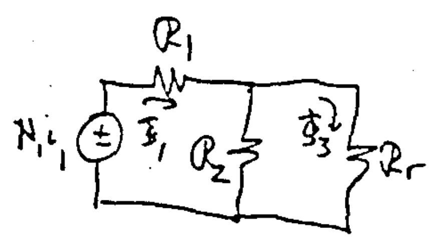
$$\Rightarrow L_{11} = \left.\frac{N_1 \Phi_1}{i_1}\right|_{i_2=0}, \quad L_{12} = \left.\frac{N_1 \Phi_1}{i_2}\right|_{i_1=0}$$

likewise, $\quad v_2 = N_2 \dot{\Phi}_3 = L_{12} \dot{i}_1 + L_{22} \dot{i}_2 \qquad$ (2nd line)

$$\Rightarrow \Phi_3 = \frac{L_{12}}{N_2} i_1 + \frac{L_{22}}{N_2} i_2$$

$$\Rightarrow L_{12} = \left.\frac{N_2 \Phi_3}{i_1}\right|_{i_2=0}, \quad L_{22} = \left.\frac{N_2 \Phi_3}{i_2}\right|_{i_1=0}$$

(i) when $i_2 = 0$, we have



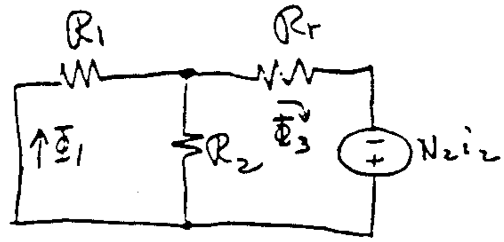
$$\left.\begin{aligned} \Phi_1 &= \frac{N_1 i_1}{\mathcal{R}_1 + \mathcal{R}_2 \| \mathcal{R}_r} \\ \Rightarrow L_{11} &= \frac{N_1^2}{\mathcal{R}_1 + \mathcal{R}_2 \| \mathcal{R}_r} \\ &= 110 \mu H \end{aligned}\right\} \text{ as for } L_1 \text{ in part (b)}$$

$$\Phi_3 = N_1 i_1 \frac{\mathcal{R}_2 \| \mathcal{R}_r}{\mathcal{R}_1 + \mathcal{R}_2 \| \mathcal{R}_r} \cdot \frac{1}{\mathcal{R}_r}$$

$\mathcal{R}_2 \| \mathcal{R}_r = 1.91 \cdot 10^5$

$$\Rightarrow L_{12} = \frac{N_1 N_2}{\mathcal{R}_1 + \mathcal{R}_2 \| \mathcal{R}_r} \cdot \frac{\mathcal{R}_2 \| \mathcal{R}_r}{\mathcal{R}_r} = 44 \mu H$$

(ii) when $i_1 = 0$, we have



$$\Phi_3 = N_2 i_2 \cdot \frac{1}{\mathcal{R}_r + \mathcal{R}_1 \| \mathcal{R}_2} \Rightarrow L_{22} = \frac{N_2^2}{\mathcal{R}_r + \mathcal{R}_1 \| \mathcal{R}_2}$$

$$= 352 \mu H$$

check:

$$\Phi_1 = N_2 i_2 \cdot \frac{\mathcal{R}_1 \| \mathcal{R}_2}{\mathcal{R}_r + \mathcal{R}_1 \| \mathcal{R}_2} \cdot \frac{1}{\mathcal{R}_1}$$

$$\Rightarrow L_{12} = \frac{N_1 N_2}{\mathcal{R}_r + \mathcal{R}_1 \| \mathcal{R}_2} \cdot \frac{\mathcal{R}_1 \| \mathcal{R}_2}{\mathcal{R}_1} = 44 \mu H \checkmark$$

so

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{d}{dt} \begin{bmatrix} 110 \mu H & 44 \mu H \\ 44 \mu H & 352 \mu H \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$