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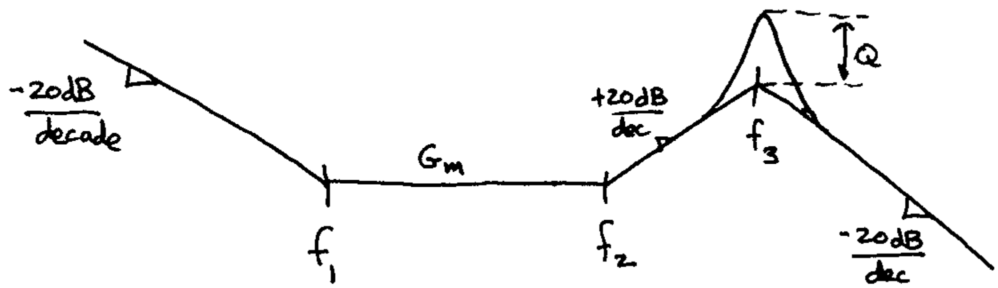
Fundamentals of Power Electronics

A Solved Problem

similar to problems 8.1–8.4



a) Express the gain represented by the asymptotes above in factored pole-zero form. f_1, f_2, and f_3 are the corner frequencies in Hz. G_m is the magnitude of the midband asymptote, as illustrated.

b) Derive analytical expressions for each asymptote, using your result from part (a).

c) Compute the value of the asymptote at $f = f_3$

Solution to (a)

We are given the magnitude of the midband asymptote G_m. So reference $G(s)$ to G_m. Poles and zeroes at frequencies $f \le f_1$ should then be expressed in inverted form.

Poles and zeroes at frequencies $f \ge f_2$ should be expressed in conventional non-inverted form.

So $G(s)$ contains the following terms;

G_m midband gain (Note that if the value of G_m is labeled in dB, then it must be converted for use in the expression for $G(s)$: $G_m = 10^{(|G_m|_{dB}/20)}$

$\left(1+\frac{\omega_1}{s}\right)$ inverted zero at $f = f_1$
$\omega_1 = 2\pi f_1$

$\left(1+\frac{s}{\omega_2}\right)$ zero at $f = f_2$
$\omega_2 = 2\pi f_2$

$\dfrac{1}{1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2}$ Complex poles at $f = f_3$
$\omega_3 = 2\pi f_3$
with Q-factor Q
(Note that if the value of Q is expressed in dB, then it must be converted for use in the expression for $G(s)$:
$Q = 10^{(|Q|_{dB}/20)}$

So

$$G(s) = G_m \frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_2}\right)}{\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)}$$

An equivalent form that does not employ inverted zeroes:

$$G(s) = G_m \frac{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}{\left(\frac{s}{\omega_1}\right)\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)}$$

b) Analytical expressions for asymptotes

Given the solution to part (a)
(either version will work):

$$G(s) = G_m \frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_2}\right)}{\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)}$$

There are four asymptotes, one for each of the following frequency ranges:

1. $f \le f_1$
2. $f_1 \le f \le f_2$
3. $f_2 \le f \le f_3$
4. $f_3 \le f$

There are three frequency-dependent terms:

$\left(1+\frac{\omega_1}{s}\right)$

$\left(1+\frac{s}{\omega_2}\right)$

$\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)$

Each of these terms consists of the sum of terms. Over a given frequency range, the asymptote is derived by neglecting the smallest term or terms within each sum, retaining only the term having the largest magnitude. This process leads to an exact expression for the asymptote, which may be a good approximation for the actual function $\|G(j\omega)\|$.

1. For $f \le f_1$

Then $f \le f_1 < f_2 < f_3$
$\omega \le \omega_1 < \omega_2 < \omega_3$

Consider each term:

$\underbrace{\left(1+\frac{\omega_1}{s}\right)}_{\text{inverted zero term}} \longrightarrow \underbrace{\frac{\omega_1}{s}}_{\text{asymptote for } f<f_1}$

justification

$$\left\| 1+\frac{\omega_1}{s} \right\|_{s=j\omega} = \sqrt{1+\left(\frac{\omega_1}{\omega}\right)^2}$$

$$\approx \sqrt{\left(\frac{\omega_1}{\omega}\right)^2} = \frac{\omega_1}{\omega}$$

since for $\omega \ll \omega_1$, $1 \ll \left(\frac{\omega_1}{\omega}\right)^2$

so $\left(1+\frac{\omega_1}{s}\right) \approx \frac{\omega_1}{s}$ for $f \ll f_1$

zero term	asymptote for $f<f_1$
$\left(1+\frac{s}{\omega_2}\right)$	1

justification

$$\left\| 1+\frac{s}{\omega_2} \right\|_{s=j\omega} = \sqrt{1+\left(\frac{\omega}{\omega_2}\right)^2}$$

$\approx \sqrt{1} = 1$ since, for $\omega \ll \omega_1$, $1 \gg \left(\frac{\omega}{\omega_2}\right)^2$. So $\left(1+\frac{s}{\omega_2}\right) \approx 1$ for $f \ll f_1$

pole term	asymptote for $f<f_1$
$\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)$	1

justification

$$\left\| 1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2 \right\| = \sqrt{\left(1-\left(\frac{\omega}{\omega_3}\right)^2\right)^2+\left(\frac{\omega}{Q\omega_3}\right)^2}$$

$\approx \sqrt{1} = 1$ since, for $\omega \ll \omega_1$,
$1 \gg \left(\frac{\omega}{\omega_3}\right)^2$ and $1 \gg \frac{\omega}{Q\omega_3}$.

so $\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right) \approx 1$ for $f \ll f_1$

Composite:
$$G(s) = G_m \frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_2}\right)}{\left(1+\frac{s}{Q\omega_3}+\left(\frac{s}{\omega_3}\right)^2\right)} \rightarrow G_m \frac{\left(\frac{\omega_1}{s}\right)(1)}{(1)} = G_m \frac{\omega_1}{s}$$

let $s=j\omega$ and take magnitude: $\left(\frac{G_m \omega_1}{\omega}\right) = \left(\frac{G_m f_1}{f}\right)$ is the expression for the magnitude asymptote for $f<f_1$.

2. for $f_1 \le f \le f_2$

Then

$$G(s) = G_m \frac{\left(1+\cancel{\frac{\omega_1}{s}}\right)\left(1+\cancel{\frac{s}{\omega_2}}\right)}{\left(1+\cancel{\frac{s}{Q\omega_3}}+\cancel{\left(\frac{s}{\omega_3}\right)^2}\right)} \longrightarrow G_m \frac{(1)(1)}{(1)} = G_m$$

3. for $f_2 \le f \le f_3$

Then

$$G(s) = G_m \frac{\left(1+\cancel{\frac{\omega_1}{s}}\right)\left(\cancel{1}+\frac{s}{\omega_2}\right)}{\left(1+\cancel{\frac{s}{Q\omega_3}}+\cancel{\left(\frac{s}{\omega_3}\right)^2}\right)} \longrightarrow G_m \frac{(1)\left(\frac{s}{\omega_2}\right)}{(1)} = G_m \frac{s}{\omega_2}$$

which has magnitude $G_m \frac{\omega}{\omega_2} = G_m \frac{f}{f_2}$

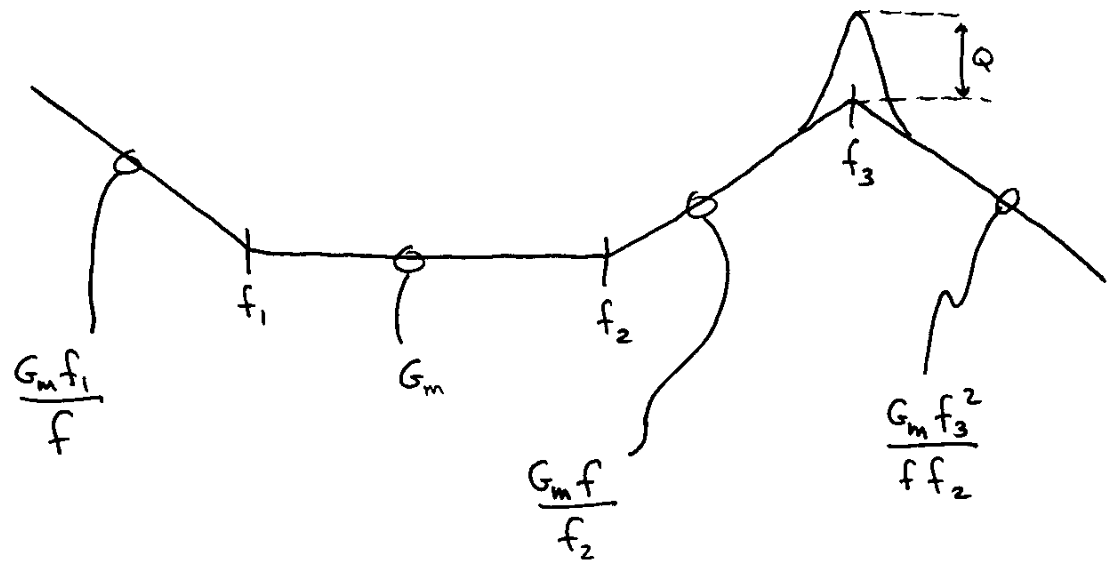
4. for $f_3 \le f$

Then

$$G(s) = G_m \frac{\left(1+\cancel{\frac{\omega_1}{s}}\right)\left(\cancel{1}+\frac{s}{\omega_2}\right)}{\left(\cancel{1}+\cancel{\frac{s}{Q\omega_3}}+\left(\frac{s}{\omega_3}\right)^2\right)} \longrightarrow G_m \frac{(1)\left(\frac{s}{\omega_2}\right)}{\left(\frac{s}{\omega_3}\right)^2} = G_m \frac{\omega_3^2}{s\omega_2}$$

which has magnitude $G_m \frac{\omega_3^2}{\omega\,\omega_2} = G_m \frac{f_3^2}{f\,f_2}$

Summary of analytical expressions for asymptotes:



c) Compute the value of the asymptote at $f = f_3$, and estimate the value of the actual magnitude.

Plug into expression for either adjacent asymptote (either the $f_2 \le f \le f_3$ or the $f_3 \le f$ asymptote).

Result is

$$\frac{G_m f_3}{f_2}$$

At $f = f_3$, the magnitude of the actual curve is given approximately by

$$\frac{Q G_m f_3}{f_2}$$

This neglects the (very small) deviation of the actual curve from the asymptotes caused by the zeroes at f_1 and f_2, but it includes the (significant) deviation caused by the Q-factor of the complex poles.